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Influence of higher-order dispersion on modulational instability and pulse broadening of partially incoherent light

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The Wigner-Moyal equation for the Wigner distribution of a partially incoherent optical wave field propagating in dispersive and nonlinear media has been generalized to include the effects of both arbitrary order of dispersion and arbitrary nonlinearity. The theory predicts partial incoherence to enhance the pulse broadening during linear wave pulse propagation. Furthermore, an application of the theory to the modulational instability of constant amplitude waves shows how higher-order dispersion affects the instability growth rate.

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I. INTRODUCTION

The propagation properties of optical pulses and beams in dispersive and nonlinear media have been a subject of intensive research for more than 40 years. An inherent assumption in most of these studies is that the optical wave is coherent. However, recently there has been considerable attention, both theoretical and experimental, given to the nonlinear propagation properties of partially incoherent light, see, e.g., Refs. [1–4]. It has been found that many of the characteristic effects associated with coherent light propagation remain, but tend to be suppressed by the partial incoherence. This is, for instance, the case with the modulational instability of continuous waves and the self-focusing collapse of two-dimensional wave beams, where typically the threshold intensity for the instability is increased by the partial incoherence; see, e.g., Refs. [5–7]. In order to describe the nonlinear dynamics of partially coherent light, several alternative methods of analysis have been used [1–4]. These methods are the mutual coherence function approach [1], the self-consistent multimode theory [2], the coherent density method [3], and the Wigner distribution function formalism [4]. The first three methods have been shown to be equivalent [8]. The fourth method, which will be used in the present work, is based on the Wigner distribution function from quantum mechanics, complemented by a Klimontovich statistical average to incorporate the coherence properties of the light, see Ref. [4] and references therein. The relation between this method and the other three has not been clear. However, we have recently shown [9] that the Wigner method is completely equivalent to the mutual coherence function approach, a result which also demonstrates the consistency of all four methods mentioned above.

In the present work we generalize the previously formulated Wigner formalism [4] to arbitrary dispersive order. In particular, in Sec. II, we derive the appropriate Wigner-Moyal equation determining the evolution of the Wigner distribution function in the presence of a nonlinearity, which is an arbitrary function of the wave intensity, and which also includes a full expansion of the linear dispersion operator. Two applications of the generalized Wigner-Moyal equation describing the influence of higher-order dispersion (in fact, third and fourth order) on partially coherent light are analyzed: in Sec. III, the dispersive broadening of wave pulses, and in Sec. IV, the modulational instability of constant amplitude waves. The result of the first problem provides, as a byproduct, a generalization of a previous classical investigation of Marcuse [10], which analyzes the effect of partial incoherence in the light source on the subsequent linear (third-order) dispersive broadening of light pulses. Conclusions are given in Sec. V.

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II. THE GENERALIZED WIGNER-MOYAL EQUATION

The classical one-dimensional nonlinear Schrödinger (NLS) equation reads

$$i\left(\frac{\partial\psi}{\partial t}+\frac{\partial\omega}{\partial k}\frac{\partial\psi}{\partial x}\right)+\frac{1}{2}\frac{\partial^2\omega}{\partial k^2}\frac{\partial^2\psi}{\partial x^2}+\kappa|\psi|^2\psi=0. \tag{1}$$

Within the classical approach, it is assumed that the nonlinear medium responds instantaneously to variations in the light intensity. This form of the NLS equation can be viewed as corresponding to an expansion of the nonlinear dispersion relation $\omega=\omega(k,|\psi|^2)$ to second dispersive order in k and to lowest nonlinear order in $|\psi|^2$. In fact, it can be seen as the lowest nontrivial expansion of the general evolution equation

$$i\frac{\partial\psi(t,x)}{\partial t}=L\left(|\psi|^2,\frac{1}{i}\frac{\vec{\partial}}{\partial x}\right)\psi(t,x), \tag{2}$$

where the L operator consists of a linear dispersion part and a nonlinear intensity dependent part, viz.,

$$L\left(|\psi|^2,\frac{1}{i}\frac{\vec{\partial}}{\partial x}\right)\equiv L_L\left(\frac{1}{i}\frac{\vec{\partial}}{\partial x}\right)+L_{NL}(|\psi|^2). \tag{3}$$

The linear dispersion operator L_L corresponds to a Taylor expansion of $\omega=\omega(k,0)$ around the carrier wave number and can be written in compact form as follows:

$$L_L\left(\frac{1}{i}\frac{\vec{\partial}}{\partial x}\right)\equiv\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\partial^n\omega}{\partial k^n}\left(\frac{1}{i}\frac{\vec{\partial}}{\partial x}\right)^n=\omega(k)\exp\left(\frac{\overleftarrow{\partial}}{\partial k}\frac{1}{i}\frac{\vec{\partial}}{\partial x}\right), \tag{4}$$

where arrows indicate the direction in which the derivatives within the operator act. The nonlinear operator is a general function of the wave intensity. When the effect of partial coherence is included in the analysis, the medium cannot respond on the (assumed) short time scale of the stochastic variations of the light field and will only experience the statistical average of the intensity denoted $\langle|\psi|^2\rangle$, cf. Ref. [4]. Thus, the nonlinear operator can be written as

$$L_{NL}(t,x)\equiv G(\langle|\psi(t,x)|^2\rangle), \tag{5}$$

which to lowest order reduces to the Kerr nonlinearity $G(x)=-\kappa x$. In this work we use the approach based on the Wigner-Moyal formalism. A general review of the application of the Wigner distribution function to partially coherent light propagation is given in Ref. [4]. This method is based on the Wigner distribution function $\rho(t,x,p)$ that in a convenient way introduces the deterministic as well as the stochastic properties of the wave through the definition

$$\rho(t,x,p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ip\xi}\langle\psi^*(t,x+\xi/2)\psi(t,x-\xi/2)\rangle d\xi. \tag{6}$$

Equation (6) implies that $\rho(t,x,p)$ and the mutual coherence function $\langle\psi^*(t,x+\xi/2)\psi(t,x-\xi/2)\rangle$ are a Fourier pair, and consequently,

$$\langle\psi^*(t,x'')\psi(t,x')\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-ip\xi}\rho(t,x,p)dp, \tag{7}$$

where for simplicity we introduce the notation $x''=x+\xi/2$, $x'=x-\xi/2$, or equivalently $x=(x'+x'')/2$, $\xi=x''-x'$. The procedure for obtaining a transport equation for the Wigner distribution [given an equation for the wave amplitude, $\psi(t,x)$] has been discussed in Refs. [11,12], but for an adaptation to the present problem and for easy reference we give the main steps of the derivation. Using Eq. (2), it is possible to rewrite the time derivative of the coherence function and in this way obtain an equation for the Wigner function $\rho(t,x,p)$. For this purpose it is convenient to use the relations

$$\frac{\partial}{\partial x'}=\frac{1}{2}\frac{\partial}{\partial x}-\frac{\partial}{\partial\xi},\quad \frac{\partial}{\partial x''}=\frac{1}{2}\frac{\partial}{\partial x}+\frac{\partial}{\partial\xi},\quad \text{and}\quad \frac{\partial}{\partial\xi}\to -ip.$$

We multiply the correspondingly rewritten Eq. (2) by $\exp(iq\xi)/(2\pi)$ and integrate over ξ. This implies that the ξ shifts of the x variables can be expressed as $\xi\to -i\partial/\partial q$. It is instructive to write out the corresponding intermediate result, which is

$$0=\frac{1}{2\pi}\int\int\left\{i\frac{\partial}{\partial t}-\left[L\left(t,x-\frac{1}{2i}\frac{\partial}{\partial q},p+\frac{1}{2i}\frac{\partial}{\partial x}\right)-L^*\left(t,x-\frac{1}{2i}\frac{\partial}{\partial q},p+\frac{1}{2i}\frac{\partial}{\partial x}\right)\right]\rho(t,x,p)e^{i\xi(q-p)}\right\}dpd\xi. \tag{8}$$

The only ξ dependence is now within the exponential and integration over this variable results in a shifted delta function, $\delta(q-p)$, which makes the second integration trivial. The result is

$$\frac{\partial\rho}{\partial t}=2\,\mathrm{Im}\left[L\left(t,x-\frac{1}{2i}\frac{\partial}{\partial p},p+\frac{1}{2i}\frac{\partial}{\partial x}\right)\rho(t,x,p)\right]. \tag{9}$$

Finally, a Taylor series expansion of the L operator around the two variables x and p completes the derivation and gives the generalized Wigner-Moyal equation:

$$\frac{\partial\rho}{\partial t}=2\,\mathrm{Im}\left\{L(t,x,p)\exp\left[\frac{i}{2}\left(\frac{\overleftarrow{\partial}}{\partial x}\frac{\vec{\partial}}{\partial p}-\frac{\overleftarrow{\partial}}{\partial p}\frac{\vec{\partial}}{\partial x}\right)\right]\rho\right\}. \tag{10}$$

In the case when the operator L is defined according to Eqs. (3)–(5), the Wigner-Moyal equation becomes

$$\frac{\partial\rho}{\partial t}+2\omega\exp\left(\frac{\overleftarrow{\partial}}{\partial k}p\right)\sin\left[\frac{1}{2}\left(\frac{\overleftarrow{\partial}}{\partial p}\frac{\vec{\partial}}{\partial x}\right)\right]\rho-2G(\langle|\psi|^2\rangle)\sin\left[\frac{1}{2}\left(\frac{\overleftarrow{\partial}}{\partial x}\frac{\vec{\partial}}{\partial p}\right)\right]\rho=0. \tag{11}$$

The averaged intensity $\langle|\psi|^2\rangle$ is expressed through Eq. (7) taken in the same point, i.e.,

$$\langle|\psi|^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\rho(t,x,p)dp. \tag{12}$$

As is well known, the NLS equation, Eq. (1), with suitably chosen evolution variable, is often used to analyze the propagation of optical pulses as well as beams. In the generalization of the linear operator part of the NLS equation, as given by Eq. (4), the coefficients $d^n\omega/dk^n$ are determined by the dispersive or diffractive properties of the medium, respectively. The second-order term in the operator L_L corresponds to the first dispersive order or paraxial approximation, which is a standard approximation in investigations of pulse and beam dynamics and at the basis of the NLS equation. However, in situations where the pulse length or the pulse width becomes sufficiently small, these approximations are not sufficient and the expansions must be carried to higher order. The importance of higher-order dispersive or diffractive effects has attracted significant interest over the years, see, e.g., Ref. [13] and references therein. We emphasize that although the analysis will be carried out with the ''timelike'' variable t as evolution variable, the analysis is equally applicable to diffraction of beams in nonlinear and noninstantaneous media.

III. NONLINEAR AND DISPERSIVE PULSE BROADENING

Solutions of the Wigner-Moyal equation are difficult to obtain analytically in the general case. However, important information about $\rho(t,x,p)$ is contained in the moments of the distribution, defined with respect to different weight functions $w(x,p)$ as follows:

$$\langle\langle w(x,p)\rangle\rangle \equiv \frac{\int\int w(x,p)\rho\, dx dp}{\int\int \rho\, dx dp}. \tag{13}$$

In particular, the moments corresponding to $w(x,p)=x$ and $w(x,p)=x^2$ have a direct physical meaning, cf. Ref. [14]. The moment $\langle\langle x\rangle\rangle$ defines the mean position and the moment $\langle\langle x^2\rangle\rangle$ determines the width of the beam. The rms width σ of a wave pulse/beam is determined by

$$\sigma^2 = \langle\langle x^2\rangle\rangle - \langle\langle x\rangle\rangle^2. \tag{14}$$

The Wigner-Moyal equation can be used to obtain information about the evolution of the moments. In particular, the second-order time derivatives of the two moments, $\langle\langle x\rangle\rangle$ and $\langle\langle x^2\rangle\rangle$, yield

$$\frac{d^2\langle\langle x\rangle\rangle}{dt^2} = -\left\langle\left\langle \sum_{n=2}^{\infty} \frac{p^{n-2}}{(n-2)!}\frac{\partial^n\omega}{\partial k^n}\frac{\partial G}{\partial x}\right\rangle\right\rangle \tag{15}$$

and

$$\frac{d^2\langle\langle x^2\rangle\rangle}{dt^2} = 2\left\langle\left\langle \left(\sum_{n=1}^{\infty}\frac{p^{n-1}}{(n-1)!}\frac{\partial^n\omega}{\partial k^n}\right)^2\right\rangle\right\rangle - 2\left\langle\left\langle \sum_{n=2}^{\infty}\frac{p^{n-2}}{(n-2)!}\frac{\partial^n\omega}{\partial k^n}x\frac{\partial G}{\partial x}\right\rangle\right\rangle. \tag{16}$$

Consider first linear propagation, i.e., $G\equiv 0$. In this case the acceleration of the mean position vanishes, irrespective of dispersive order, and $\langle\langle x\rangle\rangle$ is simply given by the linear expression $\langle\langle x\rangle\rangle = x_0 + vt$, where the initial position x_0 and the mean velocity v are given by the initial Wigner distribution function $\rho(0,x,p)$, i.e., by the properties of the initial pulse. Furthermore, it is straightforward to show that the second time derivative of $\langle\langle x^2\rangle\rangle$ is constant and that consequently the rms width must vary as a parabola in time, i.e., $\sigma^2=\sigma_0^2(1+c_1t+c_2t^2)$ with the coefficients again being determined by the initial wave form. In the coherent case, this result (to arbitrary dispersive order) was derived in Ref. [15]. To third dispersive order and for a partially coherent light source with Gaussian correlation function and Gaussian wave form, the corresponding result was derived by Marcuse [10]. The present analysis generalizes these results to arbitrary dispersive order and to arbitrary properties of the wave form and the coherence properties of the light.

In order to be explicit, we consider the case when the initial profile is a Gaussian of the form $\psi(0,x) = \sqrt{I_0}\exp[x^2/2a^2 + i\theta(x)]$. The phase function $\theta(x)$ characterizes the partial coherence of the beam as expressed by its autocorrelation function $R(\xi)$, which we also assume to be Gaussian, i.e.,

$$\langle \exp\{i[\theta(x+\xi/2)-\theta(x-\xi/2)]\}\rangle \equiv R(\xi) = \exp(-\xi^2/b^2). \tag{17}$$

This implies that the initial Wigner distribution is given by

$$\rho(0,x,p) = I_0\frac{a}{\sqrt{\pi}\Delta}\exp\{-[x^2/a^2+p^2a^2/\Delta^2]\}, \tag{18}$$

where $\Delta^2 = 1+4a^2/b^2$. We now restrict the analysis to include terms up to and including fourth-order dispersion. For this case, the constant c_1 is zero, since the pulse is symmetric around $t=0$ and for the constant c_2, we obtain

$$c_2 = \left[\frac{\beta_2^2\Delta^2}{a^4} + \frac{(\beta_3^2+2\beta_2\beta_4)\Delta^4}{4a^6} + \frac{5}{48}\frac{\beta_4^2\Delta^6}{a^8}\right], \tag{19}$$

where we have introduced the notation $\beta_n = \partial^n\omega/\partial k^n$. This result reduces to that obtained in Ref. [10] in the case where $\beta_4=0$ and to the result of Ref. [15] in the fully coherent case, when $b\to\infty$ and Δ equals unity.

The final expression for the pulse width evolution can conveniently be written in the form

$$\left[\frac{\sigma(t)}{\sigma(0)}\right]^2 = \left[1+t^2\left(\frac{\overline{\beta_2}^2}{a^4} + \frac{(\overline{\beta_3}^2+2\overline{\beta_2}\,\overline{\beta_4})}{4a^6} + \frac{5}{48}\frac{\overline{\beta_4}^2}{a^8}\right)\right], \tag{20}$$

where we have introduced the $\overline{\beta_n} = \beta_n\Delta^{n-1}$, which characterizes a fictitious effective dispersion, enhanced by the presence of the partial coherence. This result implies a simple way of accounting for the effect of the partial coherence: in the previously derived expression for the pulse broadening, the ordinary dispersion coefficients are replaced by the effective ones, the coherent ones being multiplied by the enhancement factor Δ, which is determined by the ratio of the relative widths of the pulse amplitude and the correlation function. For long coherence lengths, this ratio goes to zero and the enhancement factor approaches unity, i.e., the coherent result is regained.

The presence of higher-order dispersive terms tends to enhance the broadening of coherent wave pulses [15] (note, however, that if $\beta_2\beta_4<0$, fourth-order dispersion actually decreases the total dispersion). The partial coherence increases the pulse broadening even further.

In the case of nonlinear propagation, the rms width will vary in a nonmonotonous and nontrivial manner as a result of the interplay between linear and nonlinear effects as expressed by the first and second terms, respectively, on the right hand side of Eq. (16). This variation is difficult to describe in simple analytical terms even in the fully coherent case with only second-order dispersion and with $G(x)\sim x$, although approximate solutions have been obtained for this case [16]. The main difficulty stems from the fact that in the nonlinear case, the right hand side of Eq. (16) is no longer constant and that consequently the moment equations cannot be closed. However, in nonlinear situations, the definition of pulse width in terms of the rms value may sometimes be ambiguous. As an example of this we emphasize that for pulse propagation governed by the classical NLS equation, the rms pulse width will increase towards infinity for initially nonsoliton pulses due to dispersive radiation being shed off. This feature exists in spite of the fact that asymptotically only stationary pulses in the form of solitons appear.

On the other hand, in the two-dimensional case with $n=2$ (describing classical nonlinear diffraction of coherent light beams), the corresponding right hand side of Eq. (16) is indeed constant, the rms width of the beam is still a parabola in time and the famous virial theorem describing the possibility of beam collapse can be formulated. A generalization of this situation to include effects of partial beam coherence has recently been investigated by several authors, e.g., Refs. [6,7]. The results show the same qualitative picture as discussed above; the partial incoherence tends to increase the diffraction effect, and thus a higher beam power is needed to cause self-focusing collapse.

IV. INFLUENCE OF HIGHER-ORDER DISPERSION ON THE MODULATIONAL INSTABILITY

It has been shown that the classical modulational instability of a monochromatic stationary solution of the one-dimensional NLS equation is reduced and may even be totally suppressed for partially coherent light [4,5]. The presently extended Wigner-Moyal equation makes it possible to study the effect of higher-order dispersive terms on the modulational instability.

In order to investigate the stability of the general system to small perturbations on a partially coherent background solution, we assume the initial Wigner distribution function to be $\rho(t,x,p)=\rho_0(p)+\rho_1\exp[i(Kx-\Omega t)]$. The background distribution ρ_0 corresponds to a plane wave with a randomly varying phase. Including third- as well as fourth-order dispersion effects, the dispersion relation for the small perturbation ρ_1 can be obtained from the linearized Wigner-Moyal equation as

$$1=\frac{-\kappa}{|\beta_2|K}\int_{-\infty}^{\infty}dp\frac{\rho_0(p+K/2)-\rho_0(p-K/2)}{[p-\Omega/(\beta_2K)+H(p)]}, \quad (21)$$

where $H(p)=(\beta_3p^2-K^2\beta_3/24+\beta_4p^3/6+K^2\beta_4p/24)/\beta_2$ is the correction term due to the presence of the higher-order dispersion, β_3 and β_4.

Assuming ρ_0 in Eq. (21) to have a Lorentzian shape,

$$\rho_0(p)=\frac{\psi_0^2}{\pi}\frac{p_0}{p^2+p_0^2}, \quad (22)$$

the solution of the dispersion relation for the perturbation ρ_1 to second dispersion order, i.e., for $H(p)=0$ cf. Ref. [4] is

$$\frac{\Omega_0}{|\beta_2|K}=\frac{iK}{2}\left(\frac{4\kappa\psi_0^2}{\beta_2K^2}-1\right)^{1/2}-i,p_0, \quad (23)$$

where we have assumed that $\kappa\beta_2>0$, i.e., the case of anomalous second-order dispersion. The choice of a Lorentzian rather than a Gaussian shape of the background distribution, ρ_0, allows an exact analytical integration of Eq. (21).

Since the correction terms are small, the changes in the lowest-order dispersion relation will also be small and the instability growth can be obtained perturbatively as $\Omega=\Omega_0+\Delta\Omega$, where $|\Delta\Omega|\ll\Omega_0$. It is then straightforward to obtain expressions for $\Delta\Omega$ due to contributions from the third- and fourth-order dispersion.

A. Instability growth correction $\Delta\Omega$ due to third-order dispersion; $\beta_3\neq0$, $\beta_4\equiv0$

With β_4 equal to zero, the expression for $\Delta\Omega$ becomes purely real,

$$\frac{\Delta\Omega}{|\beta_2|K}=\frac{\beta_3}{|\beta_2|}\left[\frac{5}{24}K^2-p_0^2-\frac{p_0(K^2\beta_2-2\kappa\psi_0^2)}{K|\beta_2|\left(\frac{4\kappa\psi_0^2}{\beta_2K^2}-1\right)^{1/2}}\right]. \quad (24)$$

This result implies that third-order dispersion does not affect the instability growth rate, but only leads to a shift of the real frequency of the perturbations. In particular, in the small-K limit when $K\ll\min(p_0,K_c)$, where $K_c=2\psi_0\sqrt{\kappa/\beta_2}$, one obtains

$$\Delta\Omega=\beta_3K\left(\frac{5}{24}K^2-p_0^2-\text{sgn}(\beta_2)\frac{p_0K_c}{2}\right). \quad (25)$$

It is seen that the frequency shift may be positive or negative depending on the relative signs of β_2 and β_3 as well as on the relative magnitude of K, p_0, and K_c.

B. Instability growth correction $\Delta\Omega$ due to fourth-order dispersion; $\beta_4\neq0$, $\beta_3\equiv0$

Here, the third-order dispersion is assumed to vanish and the dominating correction term is then β_4. In this case $\Delta\Omega$ is an imaginary quantity, contributing to instability growth or alternatively damping. Explicitly written out, the fourth-order contribution becomes

$$\frac{\Delta\Omega}{|\beta_2|K}=-i\frac{\beta_4}{48|\beta_2|}\times\left[5K^2p_0-8p_0^3+\frac{2(K^2-6p_0^2)(K^2\beta_2-2\kappa\psi_0^2)}{K|\beta_2|\left(\frac{4\kappa\psi_0^2}{\beta_2K^2}-1\right)^{1/2}}\right]. \quad (26)$$

We note that even in the coherent limit [i.e., when $p_0=0$] the fourth-order dispersion term affects the instability growth in a nontrivial manner. When $p_0=0$, Eq. (26) reduces to

$$\frac{\Delta\Omega}{|\beta_2|K} = -i\frac{\beta_4}{24\beta_2}\frac{K(K^2-K_c^2/2)}{\sqrt{\frac{K_c^2}{K^2}-1}}. \tag{27}$$

This implies that the correction term can either increase or decrease the growth rate depending on the relative signs of β_2 and β_4, but also depending on whether the wave number K is larger or smaller than $K_c/\sqrt{2}$, the wave number corresponding to the maximum growth rate of the modulational instability.

The situation becomes even more complex when the partial coherence is taken into account. Consider again the small-K limit where $K \ll \min(p_0, K_c)$. The change in the growth rate then becomes

$$\frac{\Delta\Omega}{|\beta_2|K} = -i\frac{\beta_4}{24|\beta_2|}[3\,\mathrm{sgn}(\beta_2)K_c - 4p_0]. \tag{28}$$

Again the correction term tends to either enhance or suppress the instability, depending not only on the relative signs of β_2 and β_4, but also on the relative magnitude of the critical wave number K_c, as compared to the degree of coherence as expressed by p_0. As an example we note that if β_2 and β_4 are positive, the correction term increases the growth rate when $p_0 > 3K_c/4$, but decreases it in the opposite limit.

V. CONCLUSIONS

To summarize, the aim of this paper is to investigate the combined action of partial incoherence and higher-order dispersion on pulse broadening and on the modulational instability of a constant background field. We have derived a generalized equation for the Wigner distribution function that takes into account arbitrary order of dispersion and a general form of nonlinearity. Based on the moments of the distribution, which involve arbitrary dispersive order, general wave form, and coherence properties of the light, it is shown that partial incoherence enhances the pulse broadening during linear pulse propagation. It is found that the classical expressions for dispersive pulse broadening can still be used, while still including effects of partial incoherence, if a new parameter $\overline{\beta_n}$ is introduced characterizing the effective dispersion, i.e., the dispersion enhanced by the presence of partial incoherence. The analysis also shows, that third-order dispersion may be quenched by a suitable combination of second- and fourth-order dispersions. Regarding the influence of higher-order dispersion on the modulational instability, the results indicate that the odd dispersion terms do not affect the growth of the modulational instability that arises in the presence of the second-order dispersion. However, fourth-order dispersion does affect the instability growth. Depending on the relative signs of β_2 and β_4, the relative magnitude of the critical instability wave number K_c and the width of the coherence spectrum, p_0, higher-order dispersion may either enhance or weaken the instability growth.

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